

# Resonant Multilead Point-Contact Tunneling: Boundary State Formulation

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## Astract

We study a model of resonant multilead point-contact tunneling by using the boundary state formulation. At a critical point the model is described by multi-flavor chiral fermions on an infinite line with a point contact interaction at the origin. By applying the folding procedure, previously developed for the model of resonant point-contact tunneling of a single lead, we map the model onto a non-chiral fermion model defined on the half line. The resonant point-contact tunneling interaction is transcribed into a non-local effective boundary interaction in the folded setup, where the boundary state formulation is applicable. We construct the boundary states for the models with two and three leads explicitly and evaluate the correlation functions of currents operators exactly. The electron transport between the leads is dominated by the resonant point-contact tunneling in the low frequency regime. We observe some  $SU(2)$  and  $SU(3)$  group theoretical structures, which may be useful to analyze more complex models.

## 1 INTRODUCTION

In a recent paper we studied a model of resonant point-contact tunneling of a single lead [1] by using the boundary state formulation. We have shown that the correlation functions of current operators are exactly calculable and the model exhibits the essential features of the models of this type [2, 3, 4, 5, 6] in the simplest form. In the present paper we shall discuss its extensions, which have multileads, by applying the same folding procedure and the boundary state formulation. Here we only discuss the Tomonaga-Luttinger (TL)

liquids [7, 8, 9] on the leads at the critical point, so that the fermions on the leads are described by free fermion fields. In the folded setup the resonant point-contact tunneling between the leads is transcribed into a non-local boundary interaction. The main focus of the present work is the electron transport between the leads through the resonant point-contact tunneling. For this purpose we explicitly evaluate the correlation functions of the current operators in the cases of models with two and three leads. The fermion field on each lead is labeled with the flavor indices. Thus, in the folded setup, we have free  $SU(2)$  and  $SU(3)$  Dirac fermion field actions as the bulk actions for the models with two and three leads respectively. The non-local boundary interactions break these global symmetries.

The models we are about to explore in some detail may be considered as a variant of the multichannel Kondo model [10, 11, 12, 13, 14, 15], yet in a simpler form. For more general discussions of its relations to the multichannel Kondo models and others, such as the quantum Brownian motion of a particle on lattices, the reader may refer to ref.[6]. The models are initially defined on an infinite line with a resonant contact interaction in the middle in terms of chiral fermion fields. If we divide the infinite line into two half lines, and label the chiral fermion fields on each half line as the left and right movers, we can manufacture a two component Dirac fermion on a half line with a boundary interaction by folding. So the model perfectly fits into the boundary state formulation [16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26] of the closed string theory. Details of the folding procedure are given in the preceding work [1]. In the present work for the models with multileads, we apply the folding procedure to each lead separately, labeling the Dirac fermion field on each lead with flavor indices. The boundary interactions, which are non-local and bilinears of fermion fields, are not diagonal in the flavor basis. We may diagonalize the boundary interactions by choosing a new basis, which is related to the flavor basis by a  $SO(N)$  rotation transformation for the model with  $N$  leads. Constructing the boundary state in the new basis leads us to the exact calculation of the correlation functions of the current operators.

The rest of this paper is organized as follows: Section 2 deals with the resonant point-contact tunneling model with two leads. Beginning with the model in the unfolded setup, we obtain the model in the folded setup. Diagonalizing the boundary interaction and the construction of the boundary state follow from the action of the model in the folded setup. Section 3 is devoted to the resonant point-contact tunneling model with three leads. We repeat the same procedure, applied to the model with two leads, for the model with three leads. Some group theoretical structure is observed in the correlation functions of the current operators. Section 4 concludes the paper with discussions on the exact results we obtain and an extension of the present work to the resonant point-contact tunneling model with four leads.

## 2 Resonant Point-Contact Tunneling Model with Two Leads

At a critical point the one-dimensional model for the point-contact tunneling with two Tomonaga-Luttinger liquid leads may be described by the following action of two chiral fermion fields [6]

$$S = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\sigma \int d\tau \left\{ \sum_{a=1}^2 \eta^{a\dagger} (\partial_\tau + i\partial_\sigma) \eta^a + \delta(\sigma) d^\dagger \partial_\tau d + it\delta(\tau) \sqrt{2} \sum_{a=1}^2 (\eta^{a\dagger} d + \eta^a d^\dagger) \right\}. \quad (1)$$

This model may depict a resonant tunneling junction between two quantum wires. The chiral fermion fields  $\eta^a$ ,  $a = 1, 2$  are labeled with flavor indices. The degrees of freedom of the impurity, located at the origin are denoted by the Fermi fields  $d$  and  $d^\dagger$ , which are the creation and annihilation operators of charge on the resonant state. We assume that the impurity does not have an internal structure for simplicity. Since we are working on the theory at finite temperature, the theory is defined on a two dimensional Euclidean space and all Fermi fields are anti-periodic in  $\tau$ . The physical parameters are scaled appropriately so that the range of  $\tau$  is  $[0, 2\pi]$ . Since the TL liquids on the leads are at a critical point, their actions are just those of the free fermions and the coupling constant  $t$  does not get renormalized. At a non-critical point  $t$  may be replaced by a renormalized coupling constant.

### 2.1 The Model with Two Leads in the Unfolded Setup

We shall begin with the model in the unfolded setup and transcribe it into the model in the folded setup [27, 28, 29, 30]. The equations of motion for the fermion fields  $\eta^a$  in the unfolded setup follow from the action Eq.(1)

$$(\partial_\tau + i\partial_\sigma) \eta^a + it\sqrt{2}\delta(\sigma) d = 0, \quad (2a)$$

$$\partial_\tau d - it\sqrt{2} \sum_a \eta^a|_{\sigma=0} = 0, \quad a = 1, 2. \quad (2b)$$

Since the fermi fields are anti-periodic in  $\tau$  we may write

$$\eta^a(\tau, \sigma) = \sum_n \eta_n^a(\sigma) e^{in\tau}, \quad d(\tau) = \sum_n d_n e^{in\tau}, \quad n \in \mathbf{Z} + 1/2. \quad (3)$$

If we rewrite the equations of motion Eqs.(2a, 2b) in terms of normal modes, we have

$$n\eta_n^a = -\partial_\sigma \eta_n^a - t\delta(\sigma) d_n, \quad (4a)$$

$$nd_n = \frac{t}{\sqrt{2}} \sum_a [\eta_n^a(0+) + \eta_n^a(0-)]. \quad (4b)$$

From the first equation of motion, Eq.(4a) we get the continuity condition as follows

$$[\eta_n^a(0+) - \eta_n^a(0-)] + t\sqrt{2}d_n = 0. \quad (5)$$

Then it follows from Eqs.(4b,5) that

$$\left(I + \frac{t^2}{n} \mathbf{M}\right) \boldsymbol{\eta}_n(0+) = \left(I - \frac{t^2}{n} \mathbf{M}\right) \boldsymbol{\eta}_n(0-) \quad (6)$$

where  $I$  is an identity matrix and

$$\mathbf{M} = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}, \quad \boldsymbol{\eta}_n = \begin{pmatrix} \eta_n^1 \\ \eta_n^2 \end{pmatrix}. \quad (7)$$

We may diagonalize the boundary conditions Eq.(6), introducing a new basis for the Fermi fields,  $\Psi_n^a$ ,  $a = 1, 2$ , which are defined as

$$\Psi_n^1 = \frac{1}{\sqrt{2}} (\eta_n^1 + \eta_n^2), \quad \Psi_n^2 = \frac{1}{\sqrt{2}} (\eta_n^1 - \eta_n^2). \quad (8)$$

The boundary conditions are now read as

$$\Psi_n^1(0+) = \left(\frac{1 - \frac{2t^2}{n}}{1 + \frac{2t^2}{n}}\right) \Psi_n^1(0-), \quad \Psi_n^2(0+) = \Psi_n^2(0-). \quad (9)$$

As  $t$  increases from zero, the boundary condition for  $\Psi^1$  interpolates between the Neumann condition and the Dirichlet condition while the boundary condition for  $\Psi^2$  remain same as the Neumann condition.

## 2.2 The Model with Two Leads in the Folded Setup

Utilizing the folding procedure discussed in the preceding study [1] on the model with a single lead, we can easily construct the action for the model with two leads in the folded setup as

$$S_E = \frac{1}{2\pi} \int_0^\infty d\tau \int d\sigma [\bar{\Psi}^a \gamma \cdot \partial \Psi^a] + \frac{1}{2\pi} \int d\sigma \left\{ \bar{\xi} \gamma^1 \partial_\sigma \xi + i\sqrt{2}t (\bar{\Psi}^1 \gamma^1 \xi - \bar{\xi} \gamma^1 \Psi^1) \right\} \Big|_{\tau=0}, \quad (10)$$

where  $\gamma^0 = \sigma_1$ ,  $\gamma^1 = \sigma_2$ ,  $\gamma^5 = -i\gamma^0\gamma^1 = \sigma_3$ . Note that we interchange the coordinates  $\tau$  and  $\sigma$  in order to recast the model into the closed string picture, where the boundary state formulation is applicable. The chiral Fermi fields on each half line are combined together into two component Dirac spinors,  $\Psi^a$ ,  $a = 1, 2$ ,

$$\Psi^1 = \begin{pmatrix} \Psi_L^1 \\ \Psi_R^1 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \psi_L^1 + \psi_L^2 \\ \psi_R^1 + \psi_R^2 \end{pmatrix}, \quad (11a)$$

$$\Psi^2 = \begin{pmatrix} \Psi_L^2 \\ \Psi_R^2 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \psi_L^1 - \psi_L^2 \\ \psi_R^1 - \psi_R^2 \end{pmatrix}. \quad (11b)$$

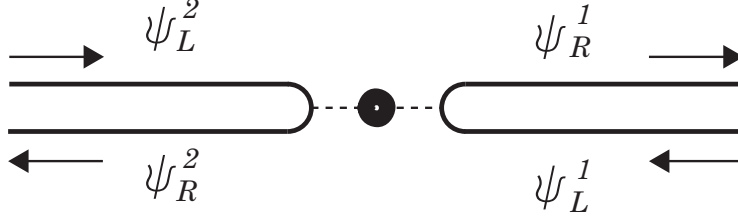


Figure 1: Resonant Point-Contact Tunneling with Two Leads in the Folded Setup

The Fermi field describing the degrees of freedom of the impurity is also extended to a two component Dirac spinor  $\xi$ .

As we integrate out the fermi fields  $\bar{\xi}$  and  $\xi$ , we obtain an effective non-local boundary action for  $\Psi^1$

$$S_{\text{Boundary}} [\bar{\Psi}^1, \Psi^1] = 2t^2 \int \frac{d\sigma}{2\pi} \bar{\Psi}^1 \gamma^1 [\partial_\sigma]^{-1} \Psi^1 \Big|_{\tau=0}. \quad (12)$$

Accordingly the boundary state can be formally written as

$$|B\rangle = \exp \left\{ 2t^2 \int \frac{d\sigma}{2\pi} \bar{\Psi}^1 \gamma^1 [\partial_\sigma]^{-1} \Psi^1 \Big|_{\tau=0} \right\} |N, N\rangle. \quad (13)$$

where the state  $|N, N\rangle$  satisfies the Neumann boundary condition for both fermi fields,  $\Psi^1$  and  $\Psi^2$ .

### 2.3 Boundary State for the Model with Two Leads

It is straightforward to find an explicit expression of the boundary state  $|B\rangle$  if we expand the two spinor components of the Fermi fields,  $\Psi_L^a$  and  $\Psi_R^a$  with the Fermion oscillators

$$\Psi_L^a(\tau + i\sigma) = \sum_n \Psi_n^a e^{-n(\tau+i\sigma)} \quad , \quad \Psi_L^{a\dagger}(\tau + i\sigma) = \sum_n \Psi_n^{a\dagger} e^{-n(\tau+i\sigma)} \quad (14a)$$

$$\Psi_R^a(\tau - i\sigma) = \sum_n \tilde{\Psi}_n^a e^{-n(\tau-i\sigma)} \quad , \quad \Psi_R^{a\dagger}(\tau - i\sigma) = \sum_n \tilde{\Psi}_n^{a\dagger} e^{-n(\tau-i\sigma)}. \quad (14b)$$

Since the Fermi fields  $\Psi_{L/R}^a$  are anti-periodic, the oscillators  $\Psi_n^a$  are labeled by half-odd-integers,  $n \in \mathbf{Z} + 1/2$ . The non-vanishing anticommutation relations between the Fermion operators are

$$\{\Psi_m^a, \Psi_n^{b\dagger}\} = \delta_{ab} \delta_{m+n} \quad , \quad \{\tilde{\Psi}_m^a, \tilde{\Psi}_n^{b\dagger}\} = \delta_{ab} \delta_{m+n}. \quad (15)$$

The vacuum state  $|0\rangle$  is annihilated by all positively moded oscillators

$$\left. \begin{aligned} \Psi_n^a |0\rangle &= 0, & \tilde{\Psi}_n^a |0\rangle &= 0 \\ \Psi_n^{a\dagger} |0\rangle &= 0, & \tilde{\Psi}_n^{a\dagger} |0\rangle &= 0 \end{aligned} \right\} \quad n > 0, \quad (16)$$

and the Neumann state  $|N, N\rangle$  satisfies

$$\Psi_n^a |N\rangle = i\tilde{\Psi}_{-n}^{a\dagger} |N\rangle, \quad \Psi_n^{a\dagger} |N\rangle = i\tilde{\Psi}_{-n}^a |N\rangle, \quad (17a)$$

$$\tilde{\Psi}_n^a |N\rangle = -i\Psi_{-n}^{a\dagger} |N\rangle, \quad \tilde{\Psi}_n^{a\dagger} |N\rangle = -i\Psi_{-n}^a |N\rangle \quad (17b)$$

where  $n$  is a positive half-odd-integer;  $n \in \mathbf{Z} + 1/2$ ,  $n > 0$ . A solution of these equations is

$$|N, N\rangle =: \exp \left\{ \sum_{a=1}^2 \sum_{n=1/2}^{\infty} i \left( \Psi_{-n}^{a\dagger} \tilde{\Psi}_{-n}^{a\dagger} + \Psi_{-n}^a \tilde{\Psi}_{-n}^a \right) \right\} : |0\rangle. \quad (18)$$

An explicit expression of the boundary state  $|B\rangle$  Eq.(13) in terms of the Fermion oscillators is found as

$$\begin{aligned} |B\rangle &= \prod_{n=1/2} : \exp \left\{ -\frac{2t^2}{n} \tilde{\Psi}_{-n}^{1\dagger} \tilde{\Psi}_n^1 \right\} :: \exp \left\{ \frac{2t^2}{n} \tilde{\Psi}_n^{1\dagger} \tilde{\Psi}_{-n}^1 \right\} : \\ &: \exp \left\{ -\frac{2t^2}{n} \Psi_{-n}^{1\dagger} \Psi_n^1 \right\} :: \exp \left\{ \frac{2t^2}{n} \Psi_n^{1\dagger} \Psi_{-n}^1 \right\} : |N, N\rangle. \end{aligned} \quad (19)$$

It can be also written as

$$\begin{aligned} |B\rangle &= \prod_{n=1/2}^{\infty} : \exp \left\{ -\frac{2t^2}{n} \Psi_{-n}^{1\dagger} \Psi_n^1 \right\} \exp \left\{ -\frac{2t^2}{n} \tilde{\Psi}_{-n}^1 \tilde{\Psi}_n^{1\dagger} \right\} : \\ &\exp \left\{ i \left( 1 - \frac{2t^2}{n} \right) \Psi_{-n}^{1\dagger} \tilde{\Psi}_{-n}^{1\dagger} \right\} \exp \left\{ i \left( 1 - \frac{2t^2}{n} \right) \Psi_{-n}^1 \tilde{\Psi}_{-n}^1 \right\} \\ &\exp \left\{ i \Psi_{-n}^{2\dagger} \tilde{\Psi}_{-n}^{2\dagger} + i \Psi_{-n}^2 \tilde{\Psi}_{-n}^2 \right\} |0\rangle. \end{aligned} \quad (20)$$

By some algebra [1], we can show that the constructed boundary state satisfies the boundary conditions for  $n = 1/2, \dots$

$$\Psi_n^1 |B\rangle = i \left( \frac{1 - 2t^2/n}{1 + 2t^2/n} \right) \tilde{\Psi}_{-n}^{1\dagger} |B\rangle, \quad (21a)$$

$$\tilde{\Psi}_n^1 |B\rangle = -i \left( \frac{1 - 2t^2/n}{1 + 2t^2/n} \right) \Psi_{-n}^{1\dagger} |B\rangle, \quad (21b)$$

$$\Psi_n^{1\dagger} |B\rangle = i \left( \frac{1 - 2t^2/n}{1 + 2t^2/n} \right) \tilde{\Psi}_{-n}^1 |B\rangle, \quad (21c)$$

$$\tilde{\Psi}_n^{1\dagger} |B\rangle = -i \left( \frac{1 - 2t^2/n}{1 + 2t^2/n} \right) \Psi_{-n}^1 |B\rangle. \quad (21d)$$

These conditions correspond to the boundary (continuity) conditions in the unfolded setup Eq.(9). The right hand sides of Eqs.(21a, 21b, 21c, 21d) differ from that of Eq.(9) by a phase factor  $\pm i$  due to the phase shift of the Fermi fields, which we adopt to transcribe the model in the unfolded setup into the model in the folded setup [1].

The boundary conditions for  $\Psi^2$  and  $\tilde{\Psi}^2$  are just the Neumann condition,

$$\Psi_n^2|B\rangle = i\tilde{\Psi}_{-n}^{2\dagger}|B\rangle, \quad (22a)$$

$$\tilde{\Psi}_n^2|B\rangle = -i\Psi_{-n}^{2\dagger}|B\rangle, \quad (22b)$$

$$\Psi_n^{2\dagger}|B\rangle = i\tilde{\Psi}_{-n}^2|B\rangle, \quad (22c)$$

$$\tilde{\Psi}_n^{2\dagger}|B\rangle = -i\Psi_{-n}^2|B\rangle. \quad (22d)$$

## 2.4 Correlation Functions of Current Operators of the Model with Two Leads

The boundary state formulation is one of the most efficient methods to evaluate correlation functions of operators. The correlation functions of the operators  $\mathcal{O}_i$ ,  $i = 1, \dots, n$ , are calculated in the boundary state formulation as

$$\langle T\mathcal{O}_1 \dots \mathcal{O}_n \rangle = \langle 0| : \mathcal{O}_1 \dots \mathcal{O}_n : |B\rangle / \langle 0|B\rangle. \quad (23)$$

Here we are interest in calculation of the correlation functions of the current operators, in particular,

$$J_{L/R}^1 = \psi_{L/R}^{1\dagger} \psi_{L/R}^1 = \frac{1}{2} \left( \Psi_{L/R}^{1\dagger} + \Psi_{L/R}^{2\dagger} \right) \left( \Psi_{L/R}^1 + \Psi_{L/R}^2 \right), \quad (24a)$$

$$J_{L/R}^2 = \psi_{L/R}^{2\dagger} \psi_{L/R}^2 = \frac{1}{2} \left( \Psi_{L/R}^{1\dagger} - \Psi_{L/R}^{2\dagger} \right) \left( \Psi_{L/R}^1 - \Psi_{L/R}^2 \right), \quad (24b)$$

from which the electron transport between the leads can be deduced. Since the boundary interaction is diagonal in the basis of  $\Psi^a$ ,  $a = 1, 2, 3$ , it may be convenient to write the current operators in this basis

$$J_L^1 = \frac{1}{2} \Psi_L^\dagger (I + \sigma^1) \Psi_L, \quad J_L^2 = \frac{1}{2} \Psi_L^\dagger (I - \sigma^1) \Psi_L, \quad (25a)$$

$$J_R^1 = \frac{1}{2} \Psi_R^\dagger (I + \sigma^1) \Psi_R, \quad J_R^2 = \frac{1}{2} \Psi_R^\dagger (I - \sigma^1) \Psi_R, \quad (25b)$$

where

$$\Psi_L = \begin{pmatrix} \Psi_L^1 \\ \Psi_L^2 \end{pmatrix}, \quad \Psi_R = \begin{pmatrix} \Psi_R^1 \\ \Psi_R^2 \end{pmatrix}. \quad (26)$$

Making use of the calculation of the correlation functions of the current operators for the case of a single lead [1] and the representations of the current operators of the model with two leads Eqs.(25a, 25b), we are able to calculate the correlation functions of the current operators. In what follows we only present the results of the calculations. The reader may refer to the preceding work [1] for some details of techniques. We expect that the correlation functions of  $\langle J_L^i(\tau, \sigma) J_L^j(\tau', \sigma') \rangle$  and  $\langle J_R^i(\tau, \sigma) J_R^j(\tau', \sigma') \rangle$ ,  $i, j = 1, 2$ , are not

affected by the boundary interaction. We may confirm it by an explicit calculation

$$\begin{aligned}\langle 0|J_L^1(\tau, \sigma)J_L^1(\tau', \sigma')|B\rangle &= \frac{1}{4} \left[ 2\frac{zz'}{(z-z')^2} + 2\frac{zz'}{(z-z')^2} \right] \\ &= \frac{zz'}{(z-z')^2},\end{aligned}\tag{27a}$$

$$\begin{aligned}\langle 0|J_L^1(\tau, \sigma)J_L^2(\tau', \sigma')|B\rangle &= \frac{1}{4} \left[ 2\frac{zz'}{(z-z')^2} - 2\frac{zz'}{(z-z')^2} \right] \\ &= 0,\end{aligned}\tag{27b}$$

$$\langle 0|J_L^2(\tau, \sigma)J_L^1(\tau', \sigma')|B\rangle = 0,\tag{27c}$$

$$\langle 0|J_L^2(\tau, \sigma)J_L^2(\tau', \sigma')|B\rangle = \frac{zz'}{(z-z')^2},\tag{27d}$$

where  $z = e^{\tau+i\sigma}$ ,  $z' = e^{\tau'+i\sigma'}$ . The correlation functions in the right moving sector are evaluated similarly

$$\langle 0|J_R^1(\tau, \sigma)J_R^1(\tau', \sigma')|B\rangle = \frac{\bar{z}\bar{z}'}{(\bar{z}-\bar{z}')^2},\tag{28a}$$

$$\langle 0|J_R^1(\tau, \sigma)J_R^2(\tau', \sigma')|B\rangle = 0,\tag{28b}$$

$$\langle 0|J_R^2(\tau, \sigma)J_R^1(\tau', \sigma')|B\rangle = 0,\tag{28c}$$

$$\langle 0|J_R^2(\tau, \sigma)J_R^2(\tau', \sigma')|B\rangle = \frac{\bar{z}\bar{z}'}{(\bar{z}-\bar{z}')^2},\tag{28d}$$

where  $\bar{z} = e^{\tau-i\sigma}$ ,  $\bar{z}' = e^{\tau'-i\sigma'}$ .

The nontrivial correlation functions are those of  $\langle J_L^i(\tau, \sigma)J_R^j(\tau', \sigma') \rangle$  and  $\langle J_R^i(\tau, \sigma)J_L^j(\tau', \sigma') \rangle$ ,  $i, j = 1, 2$ , which depict the electron transport between the leads by the resonant point-contact tunneling,

$$\begin{aligned}\langle 0|J_L^1(\tau, \sigma)J_R^1(\tau', \sigma')|B\rangle &= \frac{z\bar{z}'}{(z\bar{z}'-1)^2} - 3t^2 \frac{\sqrt{z\bar{z}'}}{z\bar{z}'-1} \ln \frac{\sqrt{z\bar{z}'}+1}{\sqrt{z\bar{z}'}-1} \\ &\quad + 4t^4 \left( \ln \frac{\sqrt{z\bar{z}'}+1}{\sqrt{z\bar{z}'}-1} \right)^2,\end{aligned}\tag{29a}$$

$$\langle 0|J_L^1(\tau, \sigma)J_R^2(\tau', \sigma')|B\rangle = -t^2 \frac{\sqrt{z\bar{z}'}}{z\bar{z}'-1} \ln \frac{\sqrt{z\bar{z}'}+1}{\sqrt{z\bar{z}'}-1} + 4t^4 \left( \ln \frac{\sqrt{z\bar{z}'}+1}{\sqrt{z\bar{z}'}-1} \right)^2,\tag{29b}$$

$$\langle 0|J_L^2(\tau, \sigma)J_R^2(\tau', \sigma')|B\rangle = \langle 0|J_L^1(\sigma)J_R^1(\sigma')|B\rangle,\tag{29c}$$

$$\langle 0|J_L^2(\tau, \sigma)J_R^1(\tau', \sigma')|B\rangle = -t^2 \frac{\sqrt{\bar{z}\bar{z}'}}{\bar{z}\bar{z}'-1} \ln \frac{\sqrt{\bar{z}\bar{z}'}+1}{\sqrt{\bar{z}\bar{z}'}-1} + 4t^4 \left( \ln \frac{\sqrt{\bar{z}\bar{z}'}+1}{\sqrt{\bar{z}\bar{z}'}-1} \right)^2.\tag{29d}$$

Note that the electron transport between different leads is solely due to the tunneling interaction, which is enhanced in the low frequency regime. As we can see that the contributions



of the tunneling interaction is long range and insensitive to the high frequency modes due to the non-local nature of the contact tunneling interaction. We may deduce the frequency dependent conductance from the correlation functions of the current operators.

### 3 Resonant Point-Contact Tunneling Model with Three Leads

Now we are ready to discuss the resonant point-contact tunneling model with three leads, which is more complex, yet more interesting. The model may be also relevant to the study of the junction of the three quantum wires or the Y-shape junction [31, 32, 33, 34, 35, 36].

#### 3.1 The Model with Three Leads in the Unfolded Setup

The resonant point-contact tunneling model with three leads is described in the unfolded setup by the same action Eq.(1) of the model with two leads, except that three chiral fermion fields  $\eta^a$ ,  $a = 1, 2, 3$ , are introduced

$$S = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\sigma \int d\tau \left\{ \sum_{a=1}^3 \eta^{a\dagger} (\partial_\tau + i\partial_\sigma) \eta^a + \delta(\sigma) d^\dagger \partial_\tau d + it\delta(\tau) \sqrt{2} \sum_{a=1}^3 (\eta^{a\dagger} d + \eta^a d^\dagger) \right\}. \quad (30)$$

The continuity condition (boundary condition) for the model with three leads is obtained as

$$\left( I + \frac{t^2}{n} \mathbf{M} \right) \boldsymbol{\eta}_n(0+) = \left( I - \frac{t^2}{n} \mathbf{M} \right) \boldsymbol{\eta}_n(0-), \quad (31a)$$

$$\mathbf{M} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \quad \boldsymbol{\eta}_n = \begin{pmatrix} \eta_n^1 \\ \eta_n^2 \\ \eta_n^3 \end{pmatrix}. \quad (31b)$$

The matrix  $\mathbf{M}$ , Eq.(31b) has a set of orthonormal eigenvectors,

$$\frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, \quad \frac{1}{\sqrt{6}} \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}, \quad (32)$$

of which eigenvalues are  $\{3, 0, 0\}$  respectively.

We may transform the matrix  $\mathbf{M}$  to a diagonal one by an  $SO(3)$  rotation

$$\mathbf{R}^t \mathbf{M} \mathbf{R} = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad (33)$$

where

$$\mathbf{R} = \begin{pmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & 0 & -\frac{\sqrt{2}}{\sqrt{3}} \end{pmatrix}, \quad \mathbf{R}^t \mathbf{R} = \mathbf{R} \mathbf{R}^t = I. \quad (34)$$

The continuity condition Eqs.(31a, 31b) is diagonalized as we introduce a new basis for the Fermi fields,  $\Psi^a$  which are related to the chiral Fermi fields  $\eta^a$  as follows

$$\Psi^a = (\mathbf{R}^t)^a_b \eta^b, \quad a, b = 1, 2, 3. \quad (35)$$

To be explicit,

$$\Psi^1 = \frac{1}{\sqrt{3}}\eta^1 + \frac{1}{\sqrt{3}}\eta^2 + \frac{1}{\sqrt{3}}\eta^3, \quad (36a)$$

$$\Psi^2 = \frac{1}{\sqrt{2}}\eta^1 - \frac{1}{\sqrt{2}}\eta^2, \quad (36b)$$

$$\Psi^3 = \frac{1}{\sqrt{6}}\eta^1 + \frac{1}{\sqrt{6}}\eta^2 - \frac{\sqrt{2}}{\sqrt{3}}\eta^3. \quad (36c)$$

The continuity conditions, which may be also understood as boundary conditions are read as follows

$$\Psi_n^1(0+) = \left( \frac{1 - \frac{3t^2}{n}}{1 + \frac{3t^2}{n}} \right) \Psi_n^1(0-), \quad \Psi_n^2(0+) = \Psi_n^2(0-), \quad \Psi_n^3(0+) = \Psi_n^3(0-). \quad (37)$$

As the coupling constant  $t$  varies, the boundary condition for  $\Psi^1$  interpolates from the Neumann condition to the Dirichlet condition.

### 3.2 The Model with Three Leads in the Folded Setup

From our study on the model with a single and two leads, we can easily construct the action for the model with three leads in the folded setup as

$$S_E = \frac{1}{2\pi} \int_0^\infty d\tau \int d\sigma \left[ \sum_{a=1}^3 \bar{\Psi}^a \gamma \cdot \partial \Psi^a \right] + \frac{1}{2\pi} \int_{\tau=0} d\sigma \left\{ \bar{\xi} \gamma^1 \partial_\sigma \xi + i\sqrt{3}t (\bar{\Psi}^1 \gamma^1 \xi - \bar{\xi} \gamma^1 \Psi^1) \right\}, \quad (38)$$

where

$$\Psi^1 = \begin{pmatrix} \Psi_L^1 \\ \Psi_R^1 \end{pmatrix} = \frac{1}{\sqrt{3}} \begin{pmatrix} \psi_L^1 + \psi_L^2 + \psi_L^3 \\ \psi_R^1 + \psi_R^2 + \psi_R^3 \end{pmatrix}, \quad (39a)$$

$$\Psi^2 = \begin{pmatrix} \Psi_L^2 \\ \Psi_R^2 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \psi_L^1 - \psi_L^2 \\ \psi_R^1 - \psi_R^2 \end{pmatrix}, \quad (39b)$$

$$\Psi^3 = \begin{pmatrix} \Psi_L^3 \\ \Psi_R^3 \end{pmatrix} = \frac{1}{\sqrt{6}} \begin{pmatrix} \psi_L^1 + \psi_L^2 - 2\psi_L^3 \\ \psi_R^1 + \psi_R^2 - 2\psi_R^3 \end{pmatrix}. \quad (39c)$$

Integrating out the fermi fields  $\bar{\xi}$  and  $\xi$ , we obtain the effective boundary action for  $\bar{\Psi}^1$  and  $\Psi^1$

$$S_{\text{eff}} [\bar{\Psi}^1, \Psi^1] = 3t^2 \int \frac{d\sigma}{2\pi} \bar{\Psi}^1 \gamma^1 [\partial_\sigma]^{-1} \Psi^1 \Big|_{\tau=0}. \quad (40)$$

Accordingly the boundary state can be written as

$$|B\rangle = \exp \left\{ 3t^2 \int \frac{d\sigma}{2\pi} \bar{\Psi}^1 \gamma^1 [\partial_\sigma]^{-1} \Psi^1 \Big|_{\tau=0} \right\} |N, N, N\rangle \quad (41)$$

where the state  $|N, N, N\rangle$  satisfies the Neumann boundary conditions for fermi fields,  $\Psi^1$ ,  $\Psi^2$  and  $\Psi^3$ .

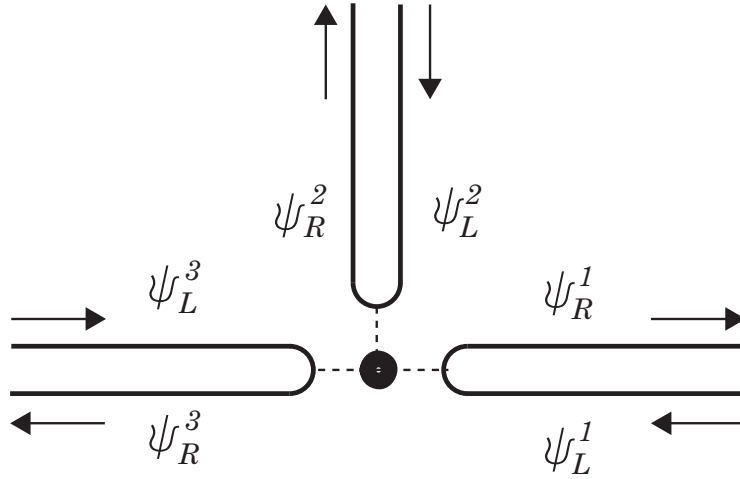


Figure 2: Resonant Point-Contact Tunneling with Three Leads in the Folded Setup

### 3.3 Boundary State for the Model with Three Leads

The boundary conditions for  $\Psi^1$  and  $\tilde{\Psi}^1$  are

$$\Psi_n^1 |B\rangle = i \left( \frac{1 - 3t^2/n}{1 + 3t^2/n} \right) \tilde{\Psi}_{-n}^{1\dagger} |B\rangle, \quad \tilde{\Psi}_n^1 = -i \left( \frac{1 - 3t^2/n}{1 + 3t^2/n} \right) \Psi_{-n}^{1\dagger} |B\rangle, \quad (42a)$$

$$\Psi_n^{1\dagger} |B\rangle = i \left( \frac{1 - 3t^2/n}{1 + 3t^2/n} \right) \tilde{\Psi}_{-n}^1 |B\rangle, \quad \tilde{\Psi}_n^{1\dagger} |B\rangle = -i \left( \frac{1 - 3t^2/n}{1 + 3t^2/n} \right) \Psi_{-n}^1 |B\rangle, \quad (42b)$$

and the Fermi field operators  $\Psi^a$  and  $\tilde{\Psi}^a$ ,  $a = 2, 3$  satisfy the Neumann condition,

$$\Psi_n^a |B\rangle = i \tilde{\Psi}_{-n}^{a\dagger} |B\rangle, \quad \tilde{\Psi}_n^a = -i \Psi_{-n}^{a\dagger} |B\rangle \quad (43a)$$

$$\Psi_n^{a\dagger} |B\rangle = i \tilde{\Psi}_{-n}^a |B\rangle, \quad \tilde{\Psi}_n^{a\dagger} |B\rangle = -i \Psi_{-n}^a |B\rangle, \quad a = 2, 3. \quad (43b)$$

These boundary conditions Eqs.(42a,42b,43a,43b) are satisfied by the constructed boundary state Eq.(41), of which expression in terms of the normal modes is given as

$$\begin{aligned}
|B\rangle &= \prod_{n=1/2}^{\infty} : \exp \left\{ -\frac{3t^2}{n} \Psi_{-n}^{1\dagger} \Psi_n^1 \right\} \exp \left\{ -\frac{3t^2}{n} \tilde{\Psi}_{-n}^1 \tilde{\Psi}_n^{1\dagger} \right\} : \\
&\exp \left\{ i \left( 1 - \frac{3t^2}{n} \right) \Psi_{-n}^{1\dagger} \tilde{\Psi}_{-n}^{1\dagger} \right\} \exp \left\{ i \left( 1 - \frac{3t^2}{n} \right) \Psi_{-n}^1 \tilde{\Psi}_{-n}^1 \right\} \\
&\prod_{a=2}^3 : \exp \left\{ i \Psi_{-n}^{a\dagger} \tilde{\Psi}_{-n}^{a\dagger} + i \Psi_{-n}^a \tilde{\Psi}_{-n}^a \right\} : |0\rangle.
\end{aligned} \tag{44}$$

### 3.4 Correlation Functions of Currents Operators of the Model with Three Leads

The current operators on each lead may be written in terms of the Fermi fields of the basis  $\{\Psi^a, a = 1, 2, 3\}$  as

$$J_L^a = \psi_L^{a\dagger} \psi_L^a = \sum_{b,c} \Psi_L^{b\dagger}(\mathbf{R}^t)_{ba}(\mathbf{R})_{ac} \Psi_L^c, \tag{45a}$$

$$J_R^a = \psi_R^{a\dagger} \psi_R^a = \sum_{b,c} \Psi_R^{b\dagger}(\mathbf{R}^t)_{ba}(\mathbf{R})_{ac} \Psi_R^c. \tag{45b}$$

It may be useful to spell out the explicit expression of the current operators for the purpose of calculating the correlation functions of the current operators

$$J_{L/R}^1 = \Psi_{L/R}^\dagger \mathbf{D}_1 \Psi_{L/R}, \quad (\mathbf{D}_1)_{ab} = (\mathbf{R})_{1a}(\mathbf{R})_{1b}, \tag{46a}$$

$$J_{L/R}^2 = \Psi_{L/R}^\dagger \mathbf{D}_2 \Psi_{L/R}, \quad (\mathbf{D}_2)_{ab} = (\mathbf{R})_{2a}(\mathbf{R})_{2b}, \tag{46b}$$

$$J_{L/R}^3 = \Psi_{L/R}^\dagger \mathbf{D}_3 \Psi_{L/R}, \quad (\mathbf{D}_3)_{ab} = (\mathbf{R})_{3a}(\mathbf{R})_{3b}, \tag{46c}$$

where

$$\mathbf{D}_1 = \begin{pmatrix} \frac{1}{3} & \frac{1}{\sqrt{6}} & \frac{1}{3\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{2} & \frac{1}{2\sqrt{3}} \\ \frac{1}{3\sqrt{2}} & \frac{1}{2\sqrt{3}} & \frac{1}{6} \end{pmatrix}, \tag{47a}$$

$$\mathbf{D}_2 = \begin{pmatrix} \frac{1}{3} & -\frac{1}{\sqrt{6}} & \frac{1}{3\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{2} & -\frac{1}{2\sqrt{3}} \\ \frac{1}{3\sqrt{2}} & -\frac{1}{2\sqrt{3}} & \frac{1}{6} \end{pmatrix}, \tag{47b}$$

$$\mathbf{D}_3 = \begin{pmatrix} \frac{1}{3} & 0 & -\frac{\sqrt{2}}{3} \\ 0 & 0 & 0 \\ -\frac{\sqrt{2}}{3} & 0 & \frac{2}{3} \end{pmatrix}. \tag{47c}$$

These are symmetric real matrices, so that they can be represented in terms of the  $SU(3)$  Gellmann matrices,  $\lambda^a$ ,  $a = 1, \dots, 8$ ,

$$\mathbf{D}_1 = \frac{1}{3}I + \frac{1}{\sqrt{6}}\lambda_1 - \frac{1}{12}\lambda_3 + \frac{1}{3\sqrt{2}}\lambda_4 + \frac{1}{2\sqrt{3}}\lambda_6 + \frac{1}{4\sqrt{3}}\lambda_8, \quad (48a)$$

$$\mathbf{D}_2 = \frac{1}{3}I - \frac{1}{\sqrt{6}}\lambda_1 - \frac{1}{12}\lambda_3 + \frac{1}{3\sqrt{2}}\lambda_4 - \frac{1}{2\sqrt{3}}\lambda_6 + \frac{1}{4\sqrt{3}}\lambda_8, \quad (48b)$$

$$\mathbf{D}_3 = \frac{1}{3}I + \frac{1}{6}\lambda_3 - \frac{\sqrt{2}}{3}\lambda_4 - \frac{1}{2\sqrt{3}}\lambda_8. \quad (48c)$$

We already know that the correlation functions of  $\langle J_L^i(\tau, \sigma) J_L^j(\tau', \sigma') \rangle$  and  $\langle J_R^i(\tau, \sigma) J_R^j(\tau', \sigma') \rangle$ ,  $i, j = 1, 2, 3$ , are not affected by the boundary interaction. But it may be worthwhile to confirm it by an explicit calculation:

$$\begin{aligned} \langle 0 | J_L^1(\sigma) J_L^1(\sigma') | B \rangle &= \left[ \frac{1}{9} + \frac{1}{4} + \frac{1}{36} + 2 \left( \frac{1}{6} + \frac{1}{18} + \frac{1}{12} \right) \right] \frac{zz'}{(z - z')^2} \\ &= \frac{zz'}{(z - z')^2}, \end{aligned} \quad (49a)$$

$$\begin{aligned} \langle 0 | J_L^1(\sigma) J_L^2(\sigma') | B \rangle &= \left[ \frac{1}{9} + \frac{1}{4} + \frac{1}{36} - 2 \left( \frac{1}{6} + \frac{1}{12} \right) + \frac{2}{9 \cdot 2} \right] \frac{zz'}{(z - z')^2} \\ &= 0. \end{aligned} \quad (49b)$$

$$\begin{aligned} \langle 0 | J_L^2(\sigma) J_L^2(\sigma') | B \rangle &= \left[ \frac{1}{9} + \frac{1}{4} + \frac{1}{36} + 2 \left( \frac{1}{6} + \frac{1}{18} + \frac{1}{12} \right) \right] \\ &= \frac{zz'}{(z - z')^2}, \end{aligned} \quad (49c)$$

$$\begin{aligned} \langle 0 | J_L^1(\sigma) J_L^3(\sigma') | B \rangle &= \left[ \frac{1}{9} + \frac{1}{6} \frac{2}{3} - 2 \frac{\sqrt{2}}{3} \frac{1}{3\sqrt{2}} \right] \frac{zz'}{(z - z')^2} \\ &= 0, \end{aligned} \quad (49d)$$

$$\begin{aligned} \langle 0 | J_L^2(\sigma) J_L^3(\sigma') | B \rangle &= \left[ \frac{1}{9} + \frac{1}{6} \frac{2}{3} - 2 \frac{\sqrt{2}}{3} \frac{1}{3\sqrt{2}} \right] \frac{zz'}{(z - z')^2} \\ &= 0, \end{aligned} \quad (49e)$$

$$\begin{aligned} \langle 0 | J_L^3(\sigma) J_L^3(\sigma') | B \rangle &= \left[ \frac{1}{9} + \frac{4}{9} + 2 \left( -\frac{\sqrt{2}}{3} \right)^2 \right] \frac{zz'}{(z - z')^2} \\ &= \frac{zz'}{(z - z')^2}. \end{aligned} \quad (49f)$$

These correlation functions are summarized as

$$\begin{aligned} \langle 0 | J_L^i(\tau, \sigma) J_L^j(\tau', \sigma') | B \rangle &= \delta^{ij} \frac{zz'}{(z - z')^2}, \\ i, j &= 1, 2, 3. \end{aligned} \quad (50)$$

The correlation functions of the right moving sector can be also calculated similarly and are succinctly summarized as

$$\begin{aligned}\langle 0|J_R^i(\tau, \sigma)J_R^j(\tau', \sigma')|B\rangle &= \delta^{ij}\frac{\bar{z}\bar{z}'}{(\bar{z}-\bar{z}')^2}, \\ i, j &= 1, 2, 3.\end{aligned}\quad (51)$$

The non-trivial correlation functions of the current operators, affected by the tunneling interaction with the impurity are  $\langle J_L^i(\tau, \sigma)J_R^j(\tau', \sigma')\rangle$ ,  $i, j = 1, 2, 3$ : The results of the explicit calculation may be summarized as

$$\langle 0|J_L^i(\tau, \sigma)J_R^j(\tau', \sigma')|B\rangle = \delta_{ij}F(z, \bar{z}') + (\mathbf{M} - I)_{ij}G(z, \bar{z}'), \quad (52a)$$

$$\begin{aligned}F(z, \bar{z}') &= \frac{z\bar{z}'}{(z\bar{z}' - 1)^2} - \frac{8}{3}t^2 \frac{\sqrt{z\bar{z}'}}{z\bar{z}' - 1} \ln \frac{\sqrt{z\bar{z}'} + 1}{\sqrt{z\bar{z}' - 1}} \\ &\quad + 4t^4 \left( \ln \frac{\sqrt{z\bar{z}'} + 1}{\sqrt{z\bar{z}' - 1}} \right)^2, \end{aligned} \quad (52b)$$

$$G(z, \bar{z}') = -\frac{2}{3}t^2 \frac{\sqrt{z\bar{z}'}}{z\bar{z}' - 1} \ln \frac{\sqrt{z\bar{z}'} + 1}{\sqrt{z\bar{z}' - 1}} + 4t^4 \left( \ln \frac{\sqrt{z\bar{z}'} + 1}{\sqrt{z\bar{z}' - 1}} \right)^2. \quad (52c)$$

The second term in Eq.(52a) describes the electron transport between the leads due to the resonant tunneling, which is dominant in the low frequency regime. The resonant tunneling interaction with the impurity suppresses the reflection currents on the leads but enhances the electron transport between different leads as the frequency is lowered.

## 4 Discussions and Conclusions

We conclude the paper with a brief discussion on the extension to the model with four leads and a remark on the boundary state. In this paper, we studied the multilead point-contact tunneling, which may be relevant to the multi-channel Kondo problem and the junctions of quantum wires, using the boundary state formulation. Although we only dealt with the models with two and three leads, our discussion can be certainly extended to the model with more than three leads. We may discuss the application of the boundary state formulation to the model with four leads briefly. The matrix  $\mathbf{M}$ , which enters the boundary conditions Eq.(31b), is replaced by a  $4 \times 4$  matrix

$$\mathbf{M} = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix}. \quad (53)$$

Its eigenvalues are  $\{4, 0, 0, 0\}$  and the corresponding orthonormal eigenvectors are respectively

$$\frac{1}{2} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \quad \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \\ 0 \\ 0 \end{pmatrix}, \quad \frac{1}{\sqrt{6}} \begin{pmatrix} 1 \\ 1 \\ -2 \\ 0 \end{pmatrix}, \quad \frac{1}{2\sqrt{3}} \begin{pmatrix} 1 \\ 1 \\ 1 \\ -3 \end{pmatrix}. \quad (54)$$

An  $SO(4)$  rotation transforms the matrix  $\mathbf{M}$  into a diagonal matrix

$$\mathbf{R}^t \mathbf{M} \mathbf{R} = \begin{pmatrix} 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad (55)$$

where

$$\mathbf{R} = \begin{pmatrix} \frac{1}{2} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & \frac{1}{2\sqrt{3}} \\ \frac{1}{2} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & \frac{1}{2\sqrt{3}} \\ \frac{1}{2} & 0 & -\sqrt{\frac{2}{3}} & \frac{1}{2\sqrt{3}} \\ \frac{1}{2} & 0 & 0 & -\frac{\sqrt{3}}{2} \end{pmatrix}, \quad \mathbf{R}^t \mathbf{R} = \mathbf{R} \mathbf{R}^t = \mathbf{I}. \quad (56)$$

The Fermi fields of the new basis  $\Psi^a$ ,  $a = 1, 2, 3, 4$ , by which the boundary interaction is diagonal are also related to the Fermi fields  $\psi^a$ ,  $a = 1, 2, 3, 4$ , on each lead (lead basis) by the  $SO(4)$  rotation

$$\Psi_{L/R}^a = \sum_{b=1}^4 (\mathbf{R}^t)_{ab} \psi_{L/R}^b. \quad (57)$$

To be explicit, we have

$$\Psi_{L/R}^1 = \frac{1}{2} \psi_{L/R}^1 + \frac{1}{2} \psi_{L/R}^2 + \frac{1}{2} \psi_{L/R}^3 + \frac{1}{2} \psi_{L/R}^4, \quad (58a)$$

$$\Psi_{L/R}^2 = \frac{1}{\sqrt{2}} \psi_{L/R}^1 - \frac{1}{\sqrt{2}} \psi_{L/R}^2, \quad (58b)$$

$$\Psi_{L/R}^3 = \frac{1}{\sqrt{6}} \psi_{L/R}^1 + \frac{1}{\sqrt{6}} \psi_{L/R}^2 - \frac{\sqrt{2}}{\sqrt{3}} \psi_{L/R}^3, \quad (58c)$$

$$\Psi_{L/R}^4 = \frac{1}{2\sqrt{3}} \psi_{L/R}^1 + \frac{1}{2\sqrt{3}} \psi_{L/R}^2 + \frac{1}{2\sqrt{3}} \psi_{L/R}^3 - \frac{\sqrt{3}}{2} \psi_{L/R}^4. \quad (58d)$$

The correlation functions of the current operators may follow from Eqs(57,23) and the boundary state for the model with four leads, which is given as

$$\begin{aligned}
|B\rangle = & \prod_{n=1/2}^{\infty} : \exp \left\{ -\frac{4t^2}{n} \Psi_{-n}^{1\dagger} \Psi_n^1 \right\} \exp \left\{ -\frac{4t^2}{n} \tilde{\Psi}_{-n}^1 \tilde{\Psi}_n^{1\dagger} \right\} : \\
& \exp \left\{ i \left( 1 - \frac{4t^2}{n} \right) \Psi_{-n}^{1\dagger} \tilde{\Psi}_{-n}^{1\dagger} \right\} \exp \left\{ i \left( 1 - \frac{4t^2}{n} \right) \Psi_{-n}^1 \tilde{\Psi}_{-n}^1 \right\} \\
& \prod_{a=2}^4 \exp \left\{ i \Psi_{-n}^{a\dagger} \tilde{\Psi}_{-n}^{a\dagger} + i \Psi_{-n}^a \tilde{\Psi}_{-n}^a \right\} |0\rangle.
\end{aligned} \tag{59}$$

Since the evaluation of the correlation functions of the current operators is straightforward, we may leave it as an exercise for the reader, who might be interested in.

Finally a remark on the boundary state concludes the paper. Following refs.[16, 18], we defined the boundary state as

$$|B\rangle = \exp(-S_{boundary}) |N\rangle. \tag{60}$$

It should be noted that the boundary interaction is treated as a perturbation. Hence the calculations based on the boundary states Eqs.(20, 44, 59) may be trusted only for small  $t$ . For large  $t$ , we may employ alternative forms of the boundary states. For an example, we may replace the boundary state Eq.(20) for the model with two leads by the following one

$$\begin{aligned}
|B\rangle = & \prod_{n=1/2}^{\infty} \exp \left\{ i \left( \frac{1 - 2t^2/n}{1 + 2t^2/n} \right) \left( \Psi_{-n}^{1\dagger} \tilde{\Psi}_{-n}^{1\dagger} + \Psi_{-n}^1 \tilde{\Psi}_{-n}^1 \right) \right\} \\
& \exp \left\{ i \left( \Psi_{-n}^{2\dagger} \tilde{\Psi}_{-n}^{2\dagger} + \Psi_{-n}^2 \tilde{\Psi}_{-n}^2 \right) \right\} |0\rangle.
\end{aligned} \tag{61}$$

This boundary state satisfies the same boundary conditions Eqs.(21a,21b,21c,21d, 22a, 22b, 22c, 22d), although it cannot be put in the form of Eq.(60). As  $t$  increases from zero to infinity, the boundary state Eq.(61) interpolates between the Neumann state and the Dirichlet state. For small  $t$ , at the leading order both expressions of the boundary states Eq.(20) and Eq.(61) agree. Further studies may be needed before employing the boundary state Eq.(61) to study the resonant point-contact tunneling at intermediate and large values of the coupling constant.

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